Time-Domain Large-Signal Modeling of Traveling-Wave Modulators on SOI

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Abstract—Silicon photonic modulators have strong nonlinear behavior in phase modulation and frequency response, which needs to be carefully addressed when they are used in high-capacity transmission systems. We demonstrate a comprehensive model for depletion-mode Mach–Zehnder modulators (MZMs) on silicon-on-insulator, which provides a bridge between device design and system performance optimization. Our methodology involves physical models of p-n-junction phase-shifters and traveling-wave electrodes, as well as circuit models for the dynamic microwave-light interactions and time-domain analysis. Critical aspects in the transmission line design for high-frequency operation are numerically studied for a case of p-n-junction loaded coplanar-strip electrode. The dynamic interaction between light and microwave is simulated using a distributed circuit model solved by the finite-difference time-domain method, allowing for accurate prediction of both small-signal and large-signal responses. The validity of the model is confirmed by the comparison with experimental results for a series push-pull MZM with a 6 mm phase shifter. The simulation shows excellent agreement with experiment for high-speed operation up to 46 Gbps. We show that this time-domain model can well predict the impact of the nonlinear behavior on the large-signal response, in contrast to the poor prediction from linear models in the frequency domain.

Index Terms—Silicon photonics, optical modulator, CMOS technology, transmission lines, small signal model, large signal model.

I. INTRODUCTION

Silicon photonics (SiP) has great potential for low-power photonic integrated systems. In particular, SiP modulators have attracted much attention for high-speed integrated transmitters [1–5]. Among various modulator structures demonstrated on silicon-on-insulator (SOI), Mach-Zehnder modulators (MZM) applying traveling-wave (TW) electrodes remain the easiest to implement for commercial optical communications systems, because they are much less thermally sensitive than resonant devices and require fewer RF connections than those using distributed drivers [4, 5]. Bit rates above 100 Gb/s have recently been achieved using SiP TW modulators with pulsed-amplitude modulation [6] and quadrature amplitude modulation (QAM) formats [7].

Despite these advances, there exist significant challenges. Compared to conventional devices on InP and LiNbO3, silicon photonic modulators have higher insertion losses and driving voltages due to the strong free-carrier absorption and relatively low electro-optic (EO) effects. Moreover, silicon photonic modulators suffer from intrinsic nonlinear effects in both optical and microwave responses. The bandwidth of a silicon modulator shows strong dependence on applied voltage. As a result, high baud-rate large signals may be seriously distorted, which is particularly challenging for complex modulation formats such as high-order QAM. The nonlinearity in the phase response causes additional challenges for higher-order modulation formats, e.g., QAM, that in general require larger phase shifts and voltage swings than simple on-off keying (OOK).

In-depth understanding and reliable modeling of the dynamics and nonlinear responses in SiP are critical for further optimization of low-power, high-speed SiP modulators and the development of advanced digital signal processing algorithms to compensate for the signal distortions of the transmitter for significantly improved transmission performance. Nevertheless, the most commonly employed models are based on small-signal analysis without considering the nonlinear characteristics [2-3]; they cannot predict time-domain responses for large-signal modulations. A reliable dynamic model that reflects both linear and nonlinear responses of SiP modulators including both electrical and optical dynamics is currently missing.

In this paper, we present and experimentally validate a comprehensive time-domain model that can accurately predict the nonlinear, large-signal response of a high-speed SiP TW modulator. We demonstrate shortcomings of the small signal model in capturing signal dynamics. Our model provides a bridge between device design and system performance optimization.

This paper is organized as follows. In section II we describe an overview of the modeling strategy. Section III presents the model for the reversed biased p-n junction phase shifter. The transmission line design of high-frequency TW electrodes are examined in section IV. Section V describes a distributed time-domain model for the dynamics in the TW modulator of section IV. In section VI, simulated results using the proposed model are compared to measured ones obtained with a device fabricated using a CMOS-compatible fabrication process. In addition, predictions from the proposed large-signal model and the existing small-signal model [2, 3] are compared, showing that the nonlinear behavior has a critical impact on high-speed data transmission. Section VII discusses how the model can predict the effect of fabrication errors and signal evolution with the device. Finally, conclusions are drawn in section VII. Several appendices provide details of our calculations.

II. OVERVIEW OF MODELLING STRATEGY

The complexity of a model capturing nonlinear effects lies in
three aspects. Firstly, high-speed optical modulation in silicon is predicated on nonlinear physical processes. Electro-optic modulation relies on the plasma effect in silicon [8], where the complex refractive index has a nonlinear relationship with carrier densities. The carrier densities in silicon p-n junctions can be modulated through carrier injection (in the forward bias condition) or depletion (in the reverse bias condition). Because the speed of carrier injection is limited by carrier lifetimes (less than 1 GHz), high-speed SiP modulators typically operate in the depletion mode. Optical modulation is achieved by varying the depletion region width, which has a nonlinear dependence on p-n junction voltage [9].

Secondly, high-frequency operation relies on high-performance traveling-wave electrodes, which requires sophisticated microwave modeling. Advanced modulation formats, such as QAM, require large phase shift. Therefore, long phase shifters, comparable to microwave wavelengths at high frequencies, are needed for high efficiency. This imposes stringent requirements on 1) velocity matching between optical waves and RF driving signals, 2) impedance matching in the microwave transmission line.

Thirdly, longitudinal distributions of optical and electrical parameters are key. Due to high RF losses, the input voltage decreases along the longitudinal (i.e., propagation) direction. Therefore, the depletion region width has a non-uniform distribution in the longitudinal direction. The resulting nonlinear distribution for optical phase responses and microwave transmission-line parameters, may significantly affect performance of large-signal modulations.

Figure 1 shows the flow chart of the model for a silicon TW modulator using a laterally doped p-n junction. This flowchart can be divided in three parts: phase shifter, traveling wave electrode, and distributed time-domain. Bulleted items (in red) are output parameters transferred as inputs to the next box.

In the phase shifter section, the carrier distributions, the distributed capacitance, $C_p$, and the distributed resistance, $R_j$, of the p-n junction are calculated as functions of the applied voltage. The calculated distributed capacitance and distributed resistance are assumed to be frequency independent.

In a separate effort, the effective index method is used to model the optical waveguide mode profile [5, 10] (lowest box in Fig. 1 left). These calculations feed into the mode overlap section (middle box in Fig. 1 left), where the changes in effective refractive index, $\Delta n_{eff}$, and optical loss due to the free carrier absorption, $\Delta \alpha$, are calculated [5, 10].

In the traveling wave (TW) electrode section, the calculated $C_j$ and $R_j$ are used to determine the required unloaded impedance for the TW electrode. In this section, we also calculate the effective index of the TW electrode, which will be used to velocity match RF and optical signals. A quasi-TEM circuit model of the unloaded transmission line inductance can be extracted from ANSYS high frequency structure simulator (HFSS) results.

Finally, the circuit model and the mode overlap results are combined to feed the distributed time-domain model in section V. The time domain model is achieved by using an FDTD numerical solution.

III. PHASE SHIFTER

A depletion-mode is assumed for high frequency operation. In Fig. 2 we present the p-n junction model including physical dimensions. Following the method used in [5, 10], we assume an abrupt junction with a uniform impurity profile in each doping region and step changes at the doping boundaries. A linear distribution in minority carrier densities is assumed between the depletion region and heavily doped region since the p/n region is much shorter than the diffusion length.

In subsection A we calculate the distributed capacitance, $C_p$, and the distributed resistance, $R_j$, of the p-n junction. Results are used in section IV to calculate the effect of the p-n-junction loaded transmission line (TML) on the complex propagation constant of the TML. In subsection B we calculate the complex propagation constant of the optical waveguide as a function of applied voltage; this parameter is used in EO modulation simulations.

A. P-N-Junction

Under reverse-bias operation, the p-n-junction can be approximated by a series combination of the resistance $R_j$ from the contact to the edge of the depletion region and the depletion capacitance $C_p$ as shown in Fig. 2 [5, 10]. The capacitance and
resistance vs. the applied driving voltage on p-n-junction, as depicted in Fig. 3a, are calculated as in [5, 10]. The levels of doping used are $5 \times 10^{17}/\text{cm}^3$ for p-dopants and $3 \times 10^{17}/\text{cm}^3$ for n-dopants as reported in [11]. Performance, EO bandwidth and impedance matching may vary due to deviation from these levels during the fabrication process. The presence of the reverse biased p-n-junction can be incorporated into the TML circuit model of a quasi TEM mode by transforming the series $R CJ$ elements into a parallel combination [12, 13]. The depletion capacitance and resistance, as shown in Fig. 3a, decrease with increasing reverse driving voltage, hence the characteristics of the loaded transmission line also become dependent on the driving voltage [5, 10, 12].

B. Mode Overlap Calculation

The plasma dispersion effect is the most efficient method of achieving optical modulation in silicon; the concentration of free charges in silicon changes the real and imaginary parts of the refractive index [5, 10]. The densities of electrons and holes (free carriers created by the p-n junction) are dependent on the optical wavelength and the applied driving voltage [5]. The analytical method explained in [5, 10] was used to find the one-dimensional (1D) field profile using the effective index method.

The changes in the refractive index from the plasma effect and the 1D field profile are required to calculate mode overlap. Using the mode overlap technique [10], we calculate the changes in effective refractive index ($\Delta n_{\text{eff}}$) and optical loss ($\alpha_{\text{loss}}$) due to free carrier distributions, plotted in Fig. 3b. $\alpha_{\text{loss}}$ is calculated by summation of the optical loss for zero voltage on p-n junction, $\alpha_0$, and variation of optical loss vs the voltage $\Delta \alpha$.

The geometrical parameters of the simulated p-n-junction are shown in Fig. 2.

IV. TW Electrode

To complete the time domain analysis in the next section, we must come up with an unloaded per unit length (p.u.l.) circuit model for the TW electrode which will be used in a circuit model of the nonlinear loaded p.u.l. TW electrode. This requires a specific design for the transmission line, which we introduce here.

Coplanar waveguide (CPW) and coplanar strip (CPS) transmission line (TML) are commonly used as traveling-wave electrodes for high-speed modulators [11-15]. The CPS is considered in this case as our traveling wave electrode because of its compact geometry in comparison to CPW. To maximize the bandwidth of a TW modulator, the following should be respected: (1) mismatch between the optical group velocity and RF group velocity should be minimized; (2) characteristic impedance of the electrode loaded by p-n-junction should match the input and output impedance of RF ports, typically 50 $\Omega$; and (3) modulator RF losses should be minimized [12, 14].

A. Group Velocity Match between RF and Optic

The RF effective index of the loaded CPS line, $n_{\text{RF,L}}$, should be equal to the group index of the optical mode, $n_{\text{g}}$, to have group velocity matched between RF and optics. The maximum unloaded microwave effective index, $n_{\text{RF,RM}}$, achievable in a CPS transmission line on SOI, without slowing down RF, is around 2.4, much lower than $n_{\text{g}}$ [12]. Because of the very low dispersion in a transmission line, the difference between RF phase velocity and RF group velocity is negligible [12, 14]. Hence we focus on slowing the RF phase velocity to force $n_{\text{RF,L}}$ to approach $n_{\text{g}}$.
The p-n-junction capacitance \( C \) loading will reduce RF phase velocity of the CPS, increasing the effective refractive index, \( n_{RF,L} \). Note that series push-pull structures, shown in Fig. 4a, have two p-n junctions in series combination, hence have only half the junction capacitance, i.e., half the slowing-down effect [1]. To further reduce the RF velocity, slow-wave electrode structures can be used [12]. We adopt the slow-wave structures seen in the inset of Fig. 4b, T-shaped extensions providing periodic capacitive loading of the CPS line [12]. The maximum effective index that can be achieved on SOI with a gap feature size of 2 \( \mu \)m is around 2.7 [12].

### B. Characteristic Impedance of TML

The unloaded CPS should be designed for impedance higher than 50 \( \Omega \) to achieve characteristic impedance of 50 \( \Omega \) when the p-n-junction is loaded. Under low-loss conditions, the required unloaded characteristic impedance, \( Z_{0,u} \), can be calculated in terms of the characteristic impedance of the loaded line, \( Z_{0,L} \), (impedance of interest) as [13]:

\[
Z_{0,u} = \frac{n_{RF,u}Z_{0,L}}{n_{RF,L}}
\]

The simulated optical group index of the rib waveguide is 3.89 near 1550 nm [5, 12] and the loaded effective index of the electrode should be equal or very close to this number for a large EO bandwidth. Using (1), these values yield a target unloaded impedance of 72 \( \Omega \) to achieve a 50 \( \Omega \) loaded impedance.

To calculated the unloaded characteristic impedance, \( Z_{0,u} \), and the effective index, \( n_{RF,u} \), a CPS transmission line is designed by HFSS while using T-shaped extensions to reduce the RF phase velocity [12, 13] (dimensions are indicated in Fig. 4b). The metal used in the HFSS simulation is assumed to be aluminum with a thickness of 2 \( \mu \)m (Metal 2 as labeled in Fig. 2). The simulated S-parameters from HFSS are converted to ABCD transfer matrix parameters using the formulations in [16] to derive the effective index and the unloaded characteristic impedance plotted in Fig. 5.

### C. RF Losses

The RF losses are related to p-n-junction resistance and unloaded electrode loss due to the skin effect of the conductor and dielectric loss. The losses from the unloaded electrode can be reduced by optimizing the geometry of the CPS as explained in [12]. The RF loss due to the semiconductor resistance can be reduced by placing higher levels of doping closer to the optical waveguide core, which, however, causes higher optical attenuation. Hence, there exists a trade-off between RF loss and optical loss. RF attenuation is dominated by p-n junctions; RF loss from the skin effect is typically negligible. A distance of 900 nm of the heavy doping from the waveguide boundary in Fig. 2 has been chosen considering the aforementioned trade-off between RF loss and optical loss.

### D. Unloaded p.u.l. Circuit Model

By assuming the quasi-TEM mode is propagating through the CPS at all frequencies of interest, the unloaded CPS can be presented by Telegrapher’s components, \( R_w, L_w, G_w, C_w \), as the unloaded p.u.l. circuit model shown in Fig. 6a. These components are calculated by the ABCD transfer matrix [16]. This model will later be used in the time-domain simulation as a part of the nonlinear CPS loaded circuit model. It should be noted that the \( R_w, L_w, G_w \) parameters are frequency dependent and the capacitance is almost constant in a wide frequency range, as

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**Fig. 5.** Simulated characteristic impedance of the CPS shown in Fig. 4 and its effective refractive index.

**Fig. 6.** (a) Distributed Telegrapher’s \( R_u, L_u, C_u, G_u \) model of the unloaded CPS, (b) distributed resistance of \( R_u \) and distributed inductance of \( L_u \), and (c) distributed conductance \( (G_u) \) and distributed capacitance \( (C_u) \).
shown in Fig. 6. The distributed conductance, \(G_n\), is very small and can be neglected in the time domain model.

The changes in the characteristic impedance and the effective index constant in the loaded CPS is described in appendix I. The loaded TML appears to be dispersive, as its microwave impedance and phase velocity are dependent on frequency and on driving voltage. The loaded characteristic impedance of the transmission line is almost constant above 1 GHz. In addition, the effective refractive index has very small variation above 20 GHz.

V. DISTRIBUTED TIME-DOMAIN MODEL

In this section, we present a time-domain distributed model describing the nonlinear relationship between the voltage induced through the RF electrode and the changes in complex effective refractive indices of RF and optical waves. The optical field behavior in the device is studied by solving the wave equation, taking into account the time and spatial variations of the optical complex refractive index induced by the modulating microwave field. A circuit-oriented large-signal model is proposed for the microwave electrodes, treated as a nonlinear transmission line.

The overall device behavior is simulated by solving, through a finite-difference approach, the coupled equations describing the electrical and optical fields propagating along the device. With the parameters obtained using the physical models of phase-shifters and the TML simulation, this model allows for dynamic large-signal analysis in the time domain.

The distributed dynamic behaviors of microwave and optical traveling fields in our model are: 1) the complex propagation constant of the optical field being locally dependent on the microwave field driving the carrier distribution and redistribution, and 2) the complex propagation constant of the TML being locally dependent on the voltage induced by the microwave field on the p-n-junction.

A. Microwave-Field Model

Here we consider a series-push-pull TW-MZM configuration, as illustrated in Fig. 4a. The electric equivalent circuit of a p.u.l. microwave electrode section is depicted in Fig. 7. In this circuit, \(L = 6.33\) nH/cm and \(C = 1.3\) pF/cm are the constant inductance and capacitance of the unloaded microwave electrode, while \(Z_{\text{con}}(f)\) models the frequency-dependent (owing to the skin effect) conductor impedance [17]. \(Z_{\text{con}}(f)\), plotted in Fig. 8, is found by adding the distributed resistance and subtracting the distributed inductance (both given in Fig. 6b) from \(L=6.33\) nH/cm (minimum value in Fig. 6b).

\[
Z_{\text{con}}(f) = R_{\text{con}} + iX_{\text{con}} = R_s + i2\pi f(L_s - 6.33\ \text{nH/cm})
\]

In this way, we are able to model the frequency dependent behavior of distributed resistance and inductance of the Telegrapher’s circuit model in time domain [17]. The series-connected p-n junctions of the two arms are modeled through the distributed semiconductor resistance, \(R_d\), and the distributed depletion capacitance \(C_d\). The series-connected junction parameters \(R_s\) and \(C_d\) are calculated by dividing \(R_s\) and \(C_s\) of Fig. 15 in appendix I by \(\Delta z\); they are voltage dependent.

The coupled voltage-current equations for the circuit shown in Fig. 7 are given by [17]:

\[
\frac{\partial i(z, t)}{\partial z} = \frac{v_{\text{con}}(z, t) - L\frac{\partial i(z, t)}{\partial t}}{R_d}
\]

\[
\frac{\partial v(z, t)}{\partial z} = \frac{v(z, t) - v_{\text{con}}(z, t)}{R_d} - C\frac{\partial v(z, t)}{\partial t}
\]

The junction microwave voltages shown in Fig. 7 are related through the following [17]

\[
v_j(z, t) = v_{\text{con}}(z, t) - R_s\left[C_d\frac{\partial v_j(z, t)}{\partial t}\right]
\]

Taking \(Z_{\text{con}}(t)\) as the inverse Fourier transform of \(Z_{\text{con}}(f)\), the voltage drop across the frequency-dependent impedance can be computed in the time-domain through the convolution [17]

\[
v_j(z, t) = Z_{\text{con}}(t) * i(z, t)
\]

When the transmission line is loaded by the p-n junction, it behaves as a new transmission line with characteristic impedance \(Z_{0L}\). The EM waves propagated through the transmission line can be divided into forward and backward waves [17]. The voltage/current relationships across the transmission line and forward–backward traveling waves are

\[
v_j(z, t) = \frac{1}{2}\left[v(z, t) - Z_{0L}i(z, t)\right]
\]
\[ v_p(z,t) = \frac{1}{2} [v(z,t) + Z_{0,L} i(z,t)] \]  
\( (8) \)

In [17], characteristic impedance of the unloaded transmission line, \( Z_{0,un} \), was used in (7) and (8). Since quantities of the induced voltage and current in the loaded and the unloaded transmission line are different, using the unloaded characteristic impedance is not sufficiently accurate for long phase shifters. Hence, we use the loaded characteristic impedance in (7) and (8).

Substituting (7) and (8) in (3) and (4), after some algebra we obtain the traveling-wave formulations

\[
\frac{\partial v_f}{\partial z} + K_0 \frac{\partial v_f}{\partial t} = -K_1 [v_f(z,t) + v_s(z,t) - v_i(z,t)] \tag{9}
\]

\[
\frac{\partial v_s}{\partial z} - K_0 \frac{\partial v_s}{\partial t} = K_1 [v_f(z,t) + v_s(z,t) - v_i(z,t)] \tag{10}
\]

where \( K_0 = 0.5 \left( \frac{Z_{0,LC} + L/Z_{0,1}}{2} \right) \), \( K_1 = Z_{0,1}/2R_0 \), \( K_2 = 1/2 \), and \( K_3 = (Z_{0,1}C - L/Z_{0,1}) \). \( K_5 \) is very small and can be neglected.

Equations (9) and (10) will be solved numerically to find the forward and backward voltages. Finally, by substituting the calculated \( v_f \) and \( v_s \) from (9) and (10) in (5), the junction voltage is found as follows

\[
v_i(z,t) = v_f(z,t) + v_s(z,t) - R_dC_d \frac{\partial v_f}{\partial t} \tag{11}
\]

The junction voltage is used for interaction between RF and optics. More details are provided in appendix II.

B. Optical-Field Model

Assuming single-mode propagation without back reflections in the optical waveguide, the optical field in the silicon waveguide is described as [17]

\[
E(x, y, z, t) = \phi(x, y) E_f(z, t) e^{i\omega t - \gamma |z|} \tag{12}
\]

where \( \phi(x, y) \) is the transverse mode distribution, the complex amplitude \( E_f(z, t) \) represents the forward slowly varying component of the optical field, \( \omega_0 \) is the laser optical frequency, and \( \gamma \) is the propagation constant in the absence of applied voltage. The amplitude can be derived from Maxwell’s equations by exploiting the slowly varying envelope approximation and treating the electric-field induced variations of the optical complex refractive index as a small perturbation [17, 18]. The time-dependent traveling-wave equation describing the optical field propagation is as follows [17, 18]

\[
\frac{\partial E_f(z,t)}{\partial z} + \frac{1}{v_p} \frac{\partial E_f(z,t)}{\partial t} = \left[ -\frac{\Delta \alpha(z,t)}{2} - i \frac{\omega_0}{c} \Delta n_{eff}(z,t) \right] E_f(z,t) \tag{13}
\]

where \( v_p \) is the group velocity, assumed to be constant over the frequency range of interest, and \( c \) is the speed of light in vacuum; \( \Delta \alpha \) and \( \Delta n_{eff} \) are the voltage-dependent changes of the optical absorption coefficient and effective refractive index in the active layer, respectively. \( \Delta \alpha \) and \( \Delta n_{eff} \) were analytically calculated in section III B and depicted in Fig. 3b. The optical propagation constant at zero bias in (12) given by

\[
\gamma_0 = \frac{1}{2} \left( \alpha_0 + \alpha_a \right) + i \frac{\omega_0}{c} n_{eff} \tag{14}
\]

\( n_{eff} \) and \( \alpha_0 \) being the effective refractive index and optical attenuation coefficient due to the free-carrier absorption at zero bias. The \( \alpha_a \) is the background propagation loss dominated by the scattering loss of waveguide sidewalls, which was estimated as \( 3 \text{ dB/cm} \) [5].

The numerical algorithm for solving the above equations is provided in detail in appendix II. They are applied to each arm of the MZM. In a push-pull operation, the two arms see opposite changes in index and attenuation (i.e., \( \Delta \alpha \) and \( \Delta n_{eff} \)), although the magnitudes are not necessary the same due to the nonlinear responses as shown in Fig. 3. The output of the MZM is then the coherent superposition of the output fields of the two arms, depending on the total phase shifts and attenuations accumulated throughout the device. By reaching this step, we are able to input driving signals to the model and observe the optical output, and examine DC and dynamic performance of the modulator, as is described in the next section.

VI. SIMULATION AND EXPERIMENTAL VALIDATION

We validate our model by predicting modulator performance and comparing predictions with experimental measurements. The simulator requires knowledge of modulator physical parameters, hence we use a modulator of our own design characterized in our lab. Performance criteria examined are: DC response vs. applied voltage, small-signal electro-optical frequency response, and large signal response in the form of eye diagrams. We contrast the eye diagrams predicted from the small signal frequency response with the large signal predictions. We show the small signal analysis greatly over estimates eye quality, in terms of rise time and extinction ratio.

A. Modulator under test

To verify the model, we consider a series push-pull TW modulator with a lateral p-n-junction operating in the depletion mode. The device examined (photograph in Fig. 9) was fabricated at IME, Singapore through CMC Microsystems. The di
o-de-waveguide structures were formed by doping rib waveguides that are 500 nm wide and 220 nm high on a 90 nm slab for operation at 1550 nm. The electrodes were fabricated in the top 2 μm thick aluminum metal layer. The interferometer was formed using low-loss compact Y-branches as the input and output couplers of the modulator. An intentional imbalance of 100 μm was incorporated to allow phase shift measurements and modulator biasing by tuning the wavelength. The device has a long phase shifter of 6 mm for low voltage. The designed doping profile parameters are explained in Fig. 2. The electrode was designed by HFSS as explained in section IV with T-shaped extensions for slowing down RF signals.

B. DC Performance

Figure 10a compares the simulated and measured $V_n$ obtained by varying the power transmission near 1550 nm by the full extinction ratio. To measure the $V_n$ of one arm, zero voltage is applied on the other arm via a short circuit; the transmission is minimized by changing the bias voltage. An external AC block was used for the DC bias, as shown in Fig. 4a. Measured $V_n$ was 6 V for arm 1 and 4.5 V for arm 2.

The difference between the two arms is most likely due to fabrication error in the doping profile. To account for this error in our time domain model, the optical phase shift, $\Delta\phi_{eff}$, due to the electro-optic modulation is multiplied by an error coefficient, $\beta_{error}$. The coefficient is varied in simulation for each arm to match the measured $V_n$. Assuming doping levels of $5e^{17}/cm^3$ (p-dopants) and $3e^{17}/cm^3$ (n-dopants), the coefficients ($\beta_{error}$) of 0.76 and 0.92 are found for the best fit for arm 1 and arm 2, respectively. This leads to higher values than the expected $V_n$ (4 V) predicted by our model, as shown in Fig. 10a. This discrepancy was also observed in others' works [12, 14]. In the subsection of large-signal analysis, effects of this error on the extinction ratio of the modulator are investigated.

C. Small signal EO Frequency Response

We measured the small-signal modulation response of the modulator up to 20 GHz (limited by the bandwidth of the vector network analyzer). The measured and simulated small-signal EO responses (EOE (dB) + 3dB) of the modulator at different bias voltages are shown in Fig. 10b. To simulate the small-signal response, a short pulse with small amplitude is transmitted through the modulator. The frequency response is then calculated by taking a fast Fourier transform. The simulated results are in good agreement with the measurement.

We can see that the 3-dB EOE bandwidth of the modulator increases from 10 GHz to 20 GHz as the reverse voltage increases from zero to 6 V, due to the reduced depletion capacitance in the p-n junction. This strong nonlinear behavior must be considered in order to correctly predict the large-signal performance. We discuss this in subsection E when comparing predictions from the nonlinear and linear models.

The bandwidth is also limited by the velocity mismatch between the RF and optical waves. As shown in Fig. 16 in appendix I, above 20 GHz, the microwave velocity of the designed modulator ranges from 3.2 to 3.8, depending on the bias voltage, which is lower than the optical group index (~3.89).

![Figure 10](image_url)

Fig. 10. (a) Optical transmission under different applied voltages, and (b) simulated and measured small-signal response at 1550 nm.
D. Eye Diagram prediction (large signal analysis)

We next simulate the large-signal modulation. To verify our model with measurement, a back-to-back link using a 66 Gbps SHF bit pattern generator (BPG), a 50 GHz SHF RF amplifier, and a digital communications analyzer (DCA) is used to study the large signal behavior of the modulator. It is important to note that an erbium doped fibre amplifier (EDFA) had to be used to reach the sensitivity of the DCA.

The optical eye diagrams of OOK modulation at different bitrates with a driving voltage of 6.5 V_{pp} at a bias of 3 V (V_{DC}) are shown in Fig. 11. Clear eye openings are observed up to 46 Gbps. Due to bandwidth limitations (about 40 GHz) of the RF connectors and probes, the simulated and measured eye diagrams at 46 Gbps are slightly different. We can see the simulation with the proposed model has good agreement with experiment.

The asymmetry in measured eye diagrams is due to the device not being exactly biased at the quadrature point, as also seen in simulation. This asymmetric behavior is related to the Mach-Zehnder interferometer (MZI) structure. Fig. 11 simulations use a raised-cosine filter with roll-off factor of one for the SHF bit pattern generator (BPG) frequency response.

E. Comparison of Small and Large Signal Models

Finally, we compare our results with the large signal performance predicted by the small signal model with a linear response assumption [2, 13]. For this comparison, we use V_{AC} = 6.5 V_{pp} and V_{DC} = 3 V for a modulator with a 6 mm phase shifter and assume no fabrication error and bias at the quadrature point. To concentrate on the modulator response, no electrical or digital filter is used.

The time-domain response simulated using the small signal model is achieved using the spectrum analysis method [2], where the output of the modulator in the time domain is obtained by taking inverse fast Fourier transform of the product of the spectrum of the driving signals and the EO frequency response at the DC bias. To calculate the EO frequency response, a frequency-domain analysis is performed following the method presented in [13]. The results match well with the small-signal analysis simulations in section VI B.

Figures 12 (a) and (b) give optical eye diagrams calculated using the large signal model and the small signal model, respectively, clearly showing that the small-signal model has significant error when the driving voltage has a large swing. The large-signal model predicts a 0–100% rise/fall time of 20.8 ps, while the small-signal model gives 4.2 ps. This significant difference is due to the nonlinear response of the modulator, ignored in the small-signal model. As shown in Fig. 10, there exists a strong dependence of the frequency response on the voltage, which is largely due to the nonlinear junction capacitance and resistance, see Fig. 3(b). The phase response and optical loss of the modulator also have nonlinear characteristics, as shown in Fig. 3(a).

The small-signal model also fails to predict the extinction ratio. The proposed large signal model predicts MZM interferometer behavior, which the small model cannot include as its calculation is based on the EO frequency response at DC bias that it is calculated based on the Fig. 15 in appendix I. In this figure, all calculations are based on the loading effect of p-n-junction in transmission line; optical parts are not involved in the small signal model. Hence, the small signal predictions are faulty.

VII. DISCUSSION

A. Modeling Effect of Fabrication Errors

To investigate the effect on extinction ratio of fabrication error, parameterized by $\beta_{error}$, we varied $\beta_{error}$ equally for both arms from one to zero while the modulator is biased at the quadrature point. Reducing $\beta_{error}$ in both arms from 1 to zero is presenting the phase shifter with lower length. From the simulated results in Fig. 13, the maximum achievable extinction ratio is 9.5 dB for 6 mm phase shifter at the quadrature point. In Fig. 13, the asymmetric error variation is also investigated. It is assumed that there is no error fabrication in one arm, $\beta_{error}=1$, and the $\beta_{error}$ of another arm is reduced towards zero. When the $\beta_{error}$ from this arm reaches zero, the modulator is acting like one single p-n-junction and the push pull operation will be removed. The simulated extinction ratio for this case is 4.3 dB.

Fig. 13. Simulated extinction ratio for 20 Gbps when $\beta_{error}$ for both arm equally are reduced and $\beta_{error}$ of one arm is fixed and another one is reduced ($\Delta \beta_{error}$).
of the phase shift becomes smaller along the electrode due to the RF propagation loss.

VIII. CONCLUSION

In summary, we have presented a comprehensive time-domain model for TW modulator on SOI. Our model incorporates a physical model for the depletion-mode phase shifter, which can well predict the nonlinear responses of critical parameters such as optical phase and loss, as well as junction capacitance and resistance. We have examined critical issues for transmission line design of TW electrodes for high-speed operation. Using the optical and electrical parameters extracted from the phase-shifter model and the numerical simulation of TW electrodes, the demonstrated distributed circuit model accurately predicts the dynamic microwave-light interactions via time-domain analysis. The model is validated experimentally with a low-voltage, TW modulator in an RF push-pull configuration applying a long CPS electrode. Excellent agreement between simulation and experiment has been obtained for both small and large signal modulations.

The comparison between a linear model and the proposed large-signal model has confirmed that the nonlinear characteristics of the silicon modulator have significant impact on the pulse shape, which have to be considered in order to accurately predict the large-signal performance. This is critical with high-capacity optical transmissions, where a large modulation depth is required and the system performance is more sensitive to the modulation quality.

While a lateral p-n junction and a CPS electrode were used in the modulator design, other types of phase-shifter configurations (e.g., vertical junction) and TW electrodes (e.g., CPW) can be readily incorporated using the same methodology. In addition, our model outputs optical fields carrying both phase and amplitude information, thus can be readily applied to advanced modulation formats, such as pulsed-amplitude modulation and quadrature amplitude modulation. With its accuracy and scalability, this model provides a powerful tool for design and optimization of silicon optical transmitters and evaluation of their performance in a system context.

**APPENDIX I**

Induced voltage and current along the propagation direction of electrometric waves in the CPS, shown in Fig. 15, when it is loaded by the p-n junction circuits can be calculated using the ABCD transmission matrix method [13]. The transmission matrix for a transmission line with unloaded characteristic impedance $Z_{0,\omega}$ propagation constant $\gamma = \alpha + i\beta$, segment length $dz_j$ for the transmission line section between $R, C, L$ elements, and $dz_k$ for the first and last sections of the transmission line, are:

$$T = \begin{bmatrix} \cosh(\gamma dz_j) & Z_{0,\omega} \sinh(\gamma dz_j) \\ \frac{1}{Z_{0,\omega}} \sinh(\gamma dz_j) & \cosh(\gamma dz_j) \end{bmatrix}$$ (15)
The effective refractive index of the loaded line can be obtained from the propagation constant as follow

$$n_l = \frac{1}{L} \Im(a \cosh(T(1,1)) \frac{c}{\omega})$$  \hspace{1cm} (20)

The loaded characteristic impedance and effective refractive index are plotted in Fig. 16. An ideal AC block (L-block in Fig. 15) was assumed in our calculation. In simulations, we chose L-block sufficiently large so that the source has an ideal impedance of 50 Ω. In experimentation, L_{block} is implemented with an external bias-tee whose DC port is connected to ground and whose RF port is connected to the RF source.

**APPENDIX II**

The numerical solution of the microwave and optical traveling-wave equations is obtained by a finite-difference approach [17, 18]. The basic idea is to longitudinally divide the device into an integer number of small sections having equal length. Across each section, the propagation characteristics of the optical and microwave waveguides are assumed to be constant, but they are allowed to change from section to section. To account for the velocity mismatch between the optical and electrical signals, two different spatial grids must be used for the optical and microwave waveguides. The finite-difference approach is schematically illustrated in Fig. 17.

The time-dependent coupled TML (9), (10) are solved in the time domain by a first-order difference approximation of the partial differentials [17, 18] yielding

$$v_f(z + \Delta z_{RF}, t + \Delta t) = (1 - k_i \Delta z_{RF}) v_f(z, t) - k_c \Delta z_{RF} v_c(z, t)$$  \hspace{1cm} (21)

$$v_c(z + \Delta z_{RF}, t + \Delta t) = -k_c \Delta z_{RF} v_c(z, t) + (1 - k_i \Delta z_{RF}) v_f(z, t) + k_j \Delta z_{RF} v_j(z, t)$$  \hspace{1cm} (22)

where we have set $\Delta z_{RF} = \Delta t/k_0$ and we have neglected the second derivatives terms.

By choosing small enough $\Delta z_{RF}$, each section acts like a lumped section; induced voltage and current across $\Delta z_{RF}$ will be constant. In this analysis the $\Delta z_{RF}$ should be smaller than wavelength/10 in the maximum frequency content of the analogue modulator bandwidth [19]. In simulations, we used 150 μm as $\Delta z_{RF}$, which respects the aforementioned criteria.

Equations (21) and (22) must be solved with the initial and boundary conditions for the forward and backward voltages at the beginning and end of the modulator. The conditions are imposed according to the applied signals to the modulator. Thus, the initial conditions for the DC bias voltage can be written as

$$V_f = V_{V0} = V_{RF}/2$$  \hspace{1cm} (23)

where we have denoted with $V_f$ and $V_r$ the dc components of the forward and backward propagating waves.

From the RF signal standpoint, the corresponding boundary conditions are

$$v_f(0, t) = v_f(0, t) = \frac{R_s - Z_{nL}}{R_s + Z_{nL}} + \frac{Z_{nL}}{R_s + Z_{nL}} v_f$$  \hspace{1cm} (24)
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